

Problem 1.1. It is known that a gas has the following properties:

- its entropy $S = aT^b$, and
- the work necessary for its isothermal compression from volume V_2 to volume V_1 is $cT \ln(V_2/V_1)$,

where a , b , and c are positive constants. Find the equation of state, thermodynamic potentials E , W , F , and G , and specific heats C_V and C_p of the gas.

Solution: From the second condition we have:

$$p(T, V_1) = -\frac{dA}{dV_1} = \frac{cT}{V_1},$$

so that the equation of state coincides with that of an ideal gas with $N = c$ (see, e.g., Eq. (42) of the lecture notes). Hence we can use Eqs. (44)-(49) of the notes to accomplish the assignment. In particular, from Eq. (44) we have

$$f'(T) = N \ln V - S = c \ln V - aT^b,$$

so that

$$f(T) = \int (c \ln V - aT^b) dT = cT \ln V - \frac{a}{b+1} T^{b+1} + d,$$

where d is a constant. Now, Eqs. (43), (45)-(49) of the lecture notes give

$$F = -NT \ln V + f(T) = -cT \ln V + f(T) = -\frac{a}{b+1} T^{b+1} + d,$$

$$E = f - Tf' = (cT \ln V - \frac{a}{b+1} T^{b+1} + d) - T(c \ln V - aT^b) = \frac{ab}{b+1} T^{b+1} + d,$$

$$W = E + pV = E + cT = \frac{ab}{b+1} T^{b+1} + cT + d,$$

$$G = F + pV = F + cT = -\frac{a}{b+1} T^{b+1} + cT + d,$$

$$\Omega = -pV = -cT,$$

$$C_V = \left(\frac{\partial E}{\partial T} \right)_V = (f - Tf')' = abT^b,$$

$$C_p = C_V + N = C_V + c = abT^b + c.$$

Note that all thermodynamic potentials but Ω are still determined up to a constant (d), and that coefficient c is irrelevant for quite a few functions (F , E , W , C_V).